בְּסִיַּעְתָּא דִשְׁמַיָּא

2020

Last edited on 9/28

2020 AP Calculus Exam

Exam Description

$1.\lim_{x\to 0}\frac{1-cos^2(2x)}{¿¿}¿=1$, using L'Hopital's

Calculations: $\frac{(-1)(-1)(2)[2\cos(2x)][\sin(2x)]}{8x}$ $\frac{4\cos(2x)\sin(2x)}{8x}=\frac{\cos(2x)\sin(2x)}{2x}$ $\frac{2\cos(2x)\cos(2x)-2\sin(2x)\sin(2x)}{2}$ $\lim_{x\to 0}\cos^2(2x)-\sin^2(2x)=1$	

2. Let f be defined. At what values of x, if any, is f **not differentiable**?

$\frac{2}{x}$	for $x \leftarrow 1$

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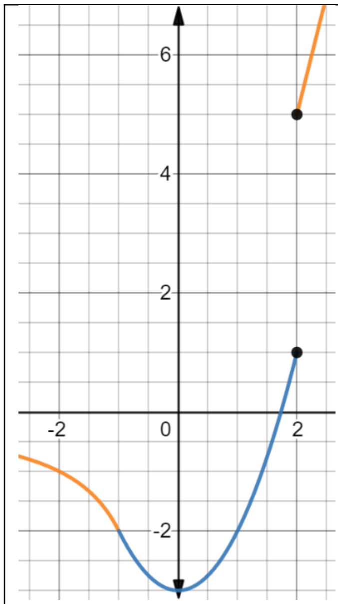
x^2-3 for $-1 \le x \le 2$
$4x-3$ for $x>2$



Analysis:
The function is differentiable at x=-1. Note that the function is not continuous at x=2. However, at x=2, they have the same slope is 4.

At x=-1, $f(x)=\frac{2}{x}$ has a hole. However, $f(x)=x^2-3$ fills the .

There is a jump discontinuity at x=2.

Calculations:

Left	Right
$f(-1)=\frac{2}{x}=\frac{2}{-1}=-2$	$f(x)=x^2-3$ $f(-1)=(-1)^2-3=1-3=-2$
Left $f(x)=2x^{-1}$ $f'(x)=-2x^{-2}$ $f'(x)=\frac{-2}{x^2}$ $f'(x)=\frac{-2}{(-1)^2}=-2$	**Right** $f(x)=x^2-3$ $f'(x)=2x$ $f'(-1)=2(-1)=-2$

Left	Right
$f(x)=x^2-3$ $f(2)=(2)^2-3=4-3=1$	$f(x)=4x-3$ $f(2)=4(2)-3=8-3=5$
Left $f(x)=x^2-3$ $f'(x)=2x$ $f'(2)=2(2)=4$	**Right** $f(x)=4x-3$ $f'(2)=4$

3. If h is the function defined by $h(x)=f(x)g(x)+2g(x)$, then $h'(1)=$¿

x	f(x)	f'(x)	g(x)	g'(x)

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1	2	-4	-5	3
2	-3	1	8	4

Calculations h(x)=f(x)g(x)+2g(x) h'(x)= f(x) g’(x)+ f’(x) g(x)+2g’(x) (2) (3) + (-4) (-5)+ 2(3) 6+20+6=32	

4. $x^3-2xy+3y^2=7$, then $\frac{dy}{dx}$

Calculations: $x^3-2xy+3y^2=7$ $3x^2-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{-3x^2+2y}{-2x+6y}$ $\frac{dy}{dx}=\frac{3x^2-2y}{2x-6y}$	

5. The radius of a right circular cylinder is **increasing** at a **rate of 2** units per second. The height of the cylinder is **decreasing** at a **rate of 5** units per second. Which of the following expressions gives the rate at which the **volume** of the cylinder is changing with respect to time in terms of the radius r and height h of the cylinder?

Calculations: Given: $\frac{dr}{dt}=2$ $\quad\frac{dh}{dt}=-5$ $\quad\frac{dv}{dt}=$ $\frac{dV}{dt}=\pi\left[r^2\frac{dh}{dt}+2r\frac{dr}{dt}h\right]$ $\frac{dV}{dt}=\pi\left[-5r^2+4rh\right]$	

6. Which of the following is equivalent to the definite integral $\int_2^6\sqrt{x}\,dx$?

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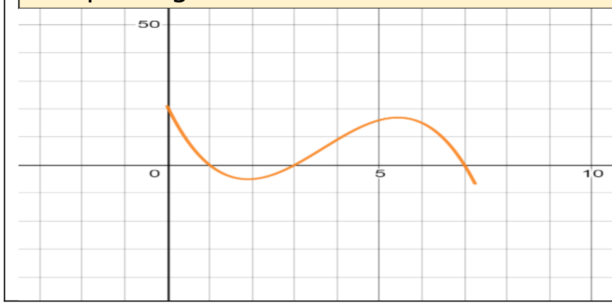
Analysis: $f\left(a+k\frac{b-a}{n}\right)\left(\frac{b-a}{n}\right)$	$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{n}\sqrt{2+\frac{4k}{n}}$ ¿

7. g is a continuous function on the interval [0,8]. Let h be the function defined by

$h(x)=\int_{3}^{x} g(t)\,dt$ On what intervals is h **increasing**?

Graph of g	**Analysis:**
	$h(x)$ is an antiderivative of g: $h(x)=\int_{0}^{x} g(t)\,dt$ $h'(x)=g(x)$

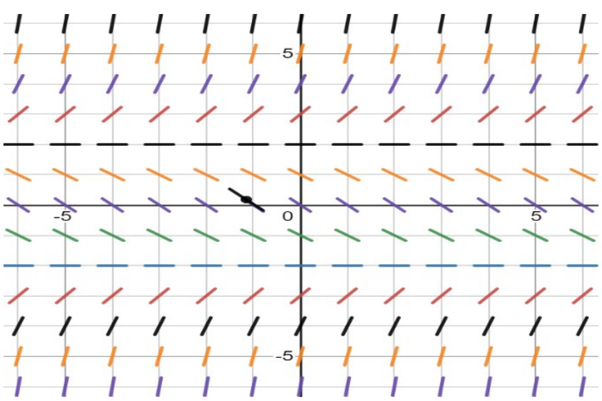
8. $\int \frac{x}{\sqrt{1-9x^2}}\,dx$

Calculations:		**Analysis:**
$u=1-9x^2$ $du=-18x\,dx$ $\frac{du}{-18x}=dx$	$-\frac{1}{18x}\int u^{-\frac{1}{2}}(x)\,du$ $-\frac{1}{18}\int u^{-\frac{1}{2}}$ $-\frac{1}{18}(2)\,u^{\frac{1}{2}}$ $-\frac{1}{9}\sqrt{1-9x^2}+c$	1. It could not be arcsine since there is an x in the numerator. u =3x and du=3 dx 2. $\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+c$ 3. It could not be arcsine since u would be u=3x and du=3. However, this radical rational expression has an x in the numerator.

9.

Regions Tested		**Analysis**:
$\frac{y-2}{2}$	$\frac{y^2-4}{2}$	
Above y=2	Above y=2	

(Positive) At $y=3$ $\frac{3-2}{2}=\frac{1}{2}$	(Positive) At $y=3$ $\frac{3^2-4}{2}=\frac{9-4}{2}=\frac{5}{2}$
At $y=1$ (Negative) $\frac{1-2}{2}=\frac{-1}{2}$	At $y=1$ (Negative) $\frac{1^2-4}{2}=\frac{1-4}{2}=\frac{-3}{2}$
At $y=-1$ (Negative) $\frac{-1-2}{2}=\frac{-3}{2}$	At $y=-1$ (Negative) $\frac{(-1)^2-4}{2}=\frac{1-4}{2}=\frac{-3}{2}$
Below y=-2 (Negative) At $y=-3$ $\frac{-3-2}{2}=\frac{-5}{2}$	Below y=-2 (Negative) At $y=-3$ $\frac{(-3)^2-4}{2}=\frac{9-4}{2}=\frac{5}{2}$



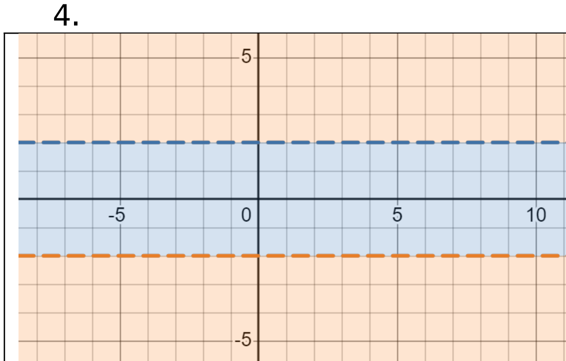
$A.\ \frac{dy}{dx}=\frac{y-2}{2}$

$B.\ \frac{dy}{dx}=\frac{y^2-4}{4}$

C. $\frac{dy}{dx}=\frac{x-2}{2}$

D. $\frac{dy}{dx}=\frac{x^2-4}{4}$

1. Choices C & D can be discarded since the slopes are independent of x.
2. At y=2, the slope should be zero. Both choice A and B satisfy this condition.
3. At y=-2, the slope should also be zero. However, at y=-2, the slope for choice A is -2 not zero. Thus, choice B is the correct answer.
4.



$\frac{3^2-4}{4}=\frac{5}{4}$	$\frac{3-2}{2}$
$\frac{0^2-4}{4}=\frac{-4}{4}=-1$	$\frac{0-2}{2}$
$\frac{(-3)^2-4}{4}=\frac{5}{4}$	$\frac{-3-2}{2}$

10. Let R be the region bounded by the graph $x=e^y$, the vertical line x=10, and the horizontal lines y=1 and y=2. Which of the following gives the **area** of R?

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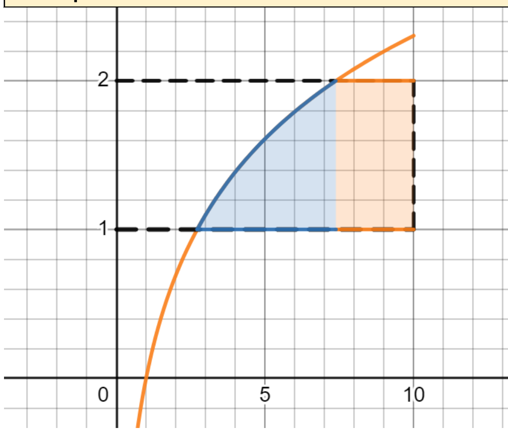
Graph of $x=e^y$



Calculations

$x=e^y$ $\qquad$ $\ln x = \ln e^y$

1. $\ln x = y \ln e$ $\qquad$ $y = \ln x$

$$\int_1^2 (10 - e^y)\, dy$$

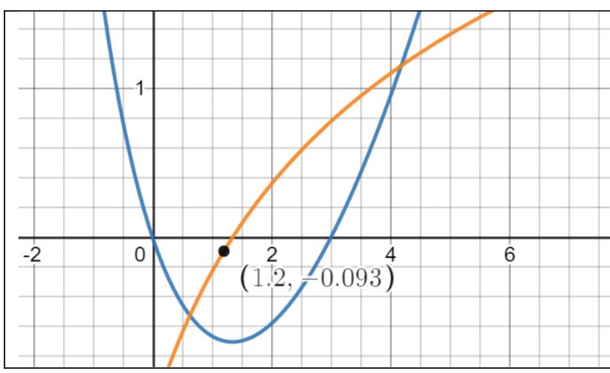
Analysis:

- The area is being calculated in terms of y.
- (Right function – left function)
- The independent variable, y, determines the limits of integration.

11. The graph of the function f is shown. The value of $\lim_{x\to 1+¿f(x)=2¿} ¿$

Analysis:	**Calculations**:
Approaches x from the right.	$\lim_{x\to 1+¿f(x)=2¿} ¿$ $\lim_{x\to 1-¿f(x)=-2¿} ¿$ Thus, $\lim_{x\to 1} f(x) = DNE$

12. The velocity of a particle moving along a straight line is given by $v(t)=1.3t\ln(.2t+.4)$ for time t≥0. What is the **acceleration** of the particle at time **t=1.2**?



$v(t) = (1.3\,t)(\ln[.2t+.4])$

$v'(t) = (1.3t)\left(\frac{0.2}{0.2t+0.4}\right) + (1.3)(\ln[0.2t+0.4])$

$a(t)=\frac{(1.3)(1.2)(.2)}{(.2)(1.2)+.4}+\frac{1.3\ln[(.2)(1.2)+.4]}{1}$

$v'(t)=\frac{.312}{.64}+1.3\ln(.64)$

a(t)= .4875-.5801732

a(t)= -.093

13. Let f be a twice-differentiable function. Values of **f'(x)**, the derivative of f, at selected values of x are given in the table. Which of the following statements are true? There exits c, where -1≤x≤-5, such that f''(c)= $\frac{-3}{2}$

x	-1	0	2	4	5
f'(x)	11	9	8	5	2

Calculations:

$\frac{9-11}{0-(-1)}=-2$	$\frac{8-9}{2-0}=\frac{-1}{2}$
$\frac{5-8}{4-2}=\frac{-3}{2}$	$\frac{2-5}{5-4}=-3$

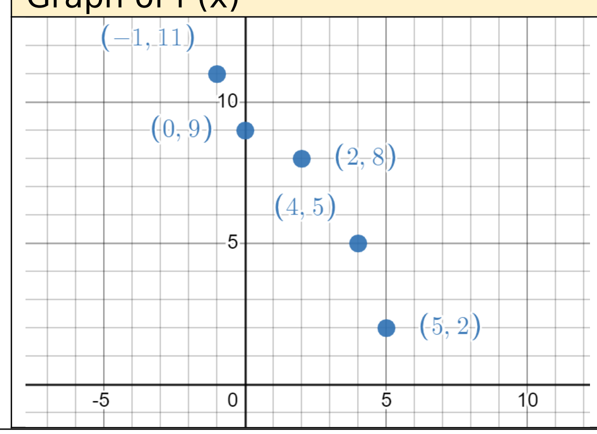
Choices:

A. f is increasing for [-1,5]
B. The graph of f is CD for -1<x<5
C. There exists c, where -1<c<5, such that f'(c)=$\frac{-3}{2}$
D. There exists c, where -1<c<5, such that f''(c)=$\frac{-3}{2}$

Analysis:

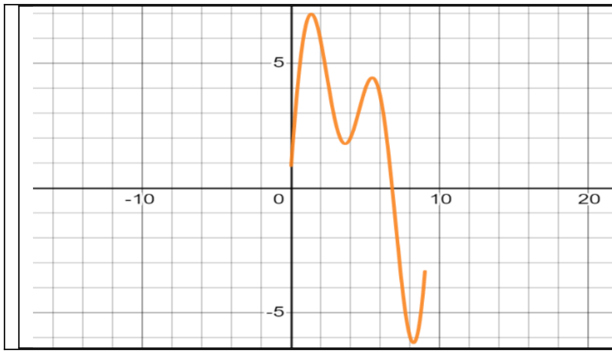
1. It cannot be discerned whether the function is increasing or decreasing from a table even thou on the table all values of f'(x)>0.
2. Concavity cannot be discerned from a table. However, since all $f''(x)<0$ (as calculated from the given table), f(x) appears to be CD.
3. Since the table contains values for f'(x) and the function is twice differentiable, then MVT can be applied to find $f''(x)$
4. The table cannot be used to find f'(x). Only f(x) can be used to find f'(x).

Graph of f'(x)



14. Let f be the function with derivative defined by $f'(x)=2+(2x-8)\sin(x+3)$. How many **POIs** does the graph of f have on the interval 0<x<9?

$f'(x)=2+(2x-8)\sin(x+3)$	**Analysis:**

15. Honey is poured through a funnel at a rate of $r(t)=4e^{-.35t}$ ounces per minute, where t is measured in minutes. How many ounces of honey are poured through the funnel from t=0 to time t=3?

Calculations: $\int_0^3 4\,e^{-.35t}$	